School Specialty
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Dear Textbook Advisory Committee,

School Specialty Math would like to take this opportunity to respond to the Indiana Board of Education in reference to why **Think Math!** should be adopted by the State of Indiana.

Included in each binder:

- **Think Math!** author Dr. E. Paul Goldenberg's response to the Dana Center's comments for Grades 3-5 regarding the Standards for Mathematical Practice
- Comments for Grades 3, 4, and 5 including information regarding the **Think Math!** Common Core Resource Guides and listings of lessons and activities, by Common Core State Standard, that demonstrate the program's coverage of the specific standard
- Complete copies of the Grades 3, 4 and 5 **Think Math!** Common Core Resource Guides

Think Math! is the newest NSF-funded mathematics curriculum program. It was created by Educational Development Center, a global innovator in math and science curriculum research, programs, strategy, and educator resources. **Think Math!** is a comprehensive Grades K-5 curriculum that combines the best of traditional, research-based, and competitive international approaches to teaching and learning math. **Think Math!** balances the acquisition of higher level processes such as problem solving and reasoning with skill fluency. The goal of **Think Math!** is true mastery for every learner.

The authors of **Think Math!,** along with other EDC staff, provided commentary during the development of the Common Core State Standards and continue to be involved in identifying and addressing issues of curriculum design to support improved teaching and learning across the country.

One of the recent additions to **Think Math!** is a Common Core Resource Guide for each grade level K-5. The Common Core Resource Guide is the **Think Math!** road map for meeting the Common Core State Standards for Mathematics. This guide includes resources that extend existing **Think Math!** chapters, deepening the program's coverage of Common Core concepts and skills.

Thank you for the opportunity to respond to the Textbook Advisory Committee's review of **Think Math!**

Sincerely,

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Think Math!

School Specialty Math
Think Math, Grades 3-5

Degree of Evidence regarding the Standards for Mathematical Practice:

Grade 3 – Moderate Evidence
Grades 4 and 5 – Minimal Evidence
Overall – Limited Evidence

Author/Publisher General Remarks:

The comments below are derived from two papers by Think Math! author Dr. E. Paul Goldenberg.

The CCSS Mathematical Practices in Think Math!
Common Core State Standards in the DNA of *Think Math!*

The two papers are also attached in full for your consideration.

Mathematical Practices in *Think Math!* pervade the entire program in age-appropriate ways; they are not handled as a separate thread or set of special lessons or supplement. The mathematical habits of mind that we articulated in 1996 became a driving principle in *all* of the EDC curriculum development work, and are central to the way we designed *Think Math!* to help students develop precisely the kind of mathematical practices described in the CCSS. Responses given here illustrate just a few examples of the ways each of the eight mathematical practices are exemplified in grades 3 through 5.

Summary of evidence:

1. **Make sense of problems and persevere in solving them**. *There is moderate evidence of this practice throughout the Grade 3 materials, but minimal evidence was cited in the Grades 4 and 5 materials. This practice is well developed and integrated in Grade 3 but not fully developed in Grades 4 and 5. The use of multiple examples of open-ended questions being used to encourage student discourse, and evidence of multiple approaches and multiple representations is well developed in the Grade 3 materials. In the Grades 4 and 5 materials, some open-ended questions were cited, but the problem-solving approach is formulated and students are not challenged to plan or justify their problem-solving approach. The use of multiple models and approaches was found to be very limited in the Grades 4 and 5 lessons.*

Author/Publisher Response:

Think Math! aims to help students make sense of *all* of the mathematics they learn. One design principle for achieving that is to put experience before formality throughout, letting students explore problems and deriving methods from the exploration. Students in upper grades, for example, learn the logic of multiplication and division—the distributive property that makes possible the algorithms we use—before the algorithms. (Algorithms are taught, but are capstones, not foundations.) The "Explore" features, throughout *Think Math!* (see hardbound Student Handbooks in Grades 3-5) exemplify that design principle, but are not the only place it is used. *Think Math!* builds stamina in problem solving both through plentiful experience with problems, but also through a unique, careful, deliberate, and pervasive development of the mathematical "infrastructure" of focus, attention, and working memory. (See http://thinkmath.edc.org/index.php/Developing_attention%2C_focus%2C_and_working_memory.)

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and

> goals. They make conjectures about the form and meaning of the solution ... rather than simply jumping into a solution attempt. —CCSS

To focus squarely on meaning and on analyzing givens and constraints, and to assure that students aren't "simply jumping into a solution attempt," *Think Math!* builds in, *every day of every year*, a kind of problem that asks *only* for the analysis and not for an "answer." This feature, "Headline Stories" (open-ended problem solving) uses several strategies. Sometimes it omits the question, requiring students to analyze the situation and decide what questions would make sense. Sometimes it omits crucial data, requiring students to consider what new information they would need. (See, e.g., Grade 4, TG p. 1061.) Of course, *Think Math!* also provides plentiful non-standard problems of the familiar variety. (See, e.g., Grade 4, TG p. 1068.)

> Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, ... Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. —CCSS

Mathematically powerful representations including the number line, area model, and tabular/spreadsheet forms are taught, as a kind of common language, to students. They use these appropriately and *show* their understanding of the correspondence between symbolic forms and other representations. *Think Math!* also supports the development of appropriate academic and mathematical language—information for the teacher is supplied in the TG for each lesson—and encourages question and discussion in class.

2. **Reason abstractly and quantitatively**. *There is inconsistent evidence to support this practice throughout this grade span. There is limited evidence of this practice being developed in Grade 3, but the evidence is minimal in the Grades 4 and 5 materials. Evidence of representing scenarios symbolically and applying understanding, not just using algorithms, was found repeatedly in the Grade 3 materials. Students in Grades 4 and 5 use only the standard algorithm and are not asked to reason abstractly or symbolically. Very little evidence was found to suggest students consider reasonableness.*

Author/Publisher Response:

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. —CCSS

In Grade 4, chapter 3, students are exploring regrouping in a new way in preparation for formalizing their understanding of place value for computation. Children work with a context, an eraser factory that groups 7 erasers to a package, 7 packages to a box. They *decontextualize* as they study their shipments to assure the no-more-than-6 rule is observed; and they *contextualize* to make sense of the trades and to work out how many erasers in a particular size container. They must constantly be attending to the units involved and the meaning of quantities. The choice of 7 practices their 7-facts, but the students' focus is on regrouping.

3. **Construct viable arguments and critique the reasoning of others**. *There was moderate evidence found of this practice in the Grade 3 materials, but minimal evidence was found in the Grades 4 and 5 samples. Reviewers for Grade 3 cited evidence of student discourse involving justifying conclusions and explaining problem-solving methods. Grades 4 and 5 materials suggest some problem solving using different assumptions, but there is a lack of evidence of student arguing, justifying, or critiquing others.*

Author/Publisher Response:

> … Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. … Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. —CCSS

Think Math! students are involved, in all grades, in age-appropriate proof and justification—as means for explaining what they have done, seen, or figured out. Grade 5, Chapter 2, lessons 8-11 are a particularly good illustration. Students perform a multiplication experiment from which they derive an unexpected pattern—the square of a number (e.g., 8×8) will always be one greater than the product of that number's neighbors (e.g., 7×9)—they are challenged to describe the pattern, extend the pattern, and even investigate a proof (using the area model) for the pattern. Students develop posters that demonstrate their proof, and explain their understanding to each other.

Think Math! pedagogy encourages discussion, listening to others, and asking useful questions, but does *not* generally ask young students to critique the reasoning of others, as it is often too hard for young students to distinguish flaws in the logic of another student's argument from artifacts created by the difficulty all young students have in articulating their thinking without ambiguity.

4. **Model with mathematics.** *There was limited evidence found for this practice, and it is underdeveloped throughout this series. Some evidence was cited in the Grade 3 materials for using and creating mathematical models/tools, but in the Grades 4 and 5 materials evidence of students creating and revising mathematical tools was not found. Minimal evidence of answering problems in context was found across the grade span, but overall this practice is underdeveloped.*

Author/Publisher Response:

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life…. In early grades, this might be as simple as writing an addition equation to describe a situation. …. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation…. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs…. They…reflect on whether the results make sense…. —CCSS

The problem "Sam Houston Elementary School has nearly 1,000 children from kindergarten through 5th grade, with about the same number of students in each grade. Most classes have close to 25 students. No class has more than 25 students. What can you figure out from this information?" appears in grade 5, TG pg 647. It asks students to work with approximations, identify the important quantities and figure out their relationships, make sense (standard 1) and check whether their results make sense. Problems of this kind, age-appropriate complexity and mathematical demand, are quite like the problems of real life, with some loose ends, and not necessarily a clear starting place. They, along with more familiar problem types, appear throughout *Think Math!*

Most of the work of Headline Stories is aimed at having students find and express the mathematics in a situation where information is given, but the question is not. E.g., Grade 4 Chapter 3 Lesson 3 (TG161) "Jin has balancing scales and weights of 1, 2, 4, and 8 ounces." If a targeted question were provided—like "What weights would she use to balance 11 ounces of butter?"—students would get arithmetic practice, but nothing more: as soon as the solution is found, it's all over. A looser question "What quantities of butter can she weigh exactly using one or more of these weights?" gives more practice, and more opportunities to think and puzzle. Over time, students get used to the generic question: What (mathematical statements) can you say *for sure* about this situation? What mathematical observations can you make? What mathematical questions does this raise? What can you figure out?

5. **Use appropriate tools strategically.** *There is minimal evidence for support of this practice, and it is a particular weakness of this series. Reviewers cited a few examples of students selecting tools to solve problems in the Grade 3, but no examples were cited in the Grades 4 and 5 materials. There is no evidence of student opportunities to use tools appropriately and strategically or to evaluate tools for strengths or limitations within this resource.*

Author/Publisher Response:

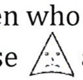
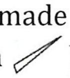
> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet.... Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. —CCSS

The variety of tools that *Think Math!* students acquire and become proficient with—including visual models such as number line or area models, mental work, or paper and pencil to support calculations—is described in the paper "Common Core State Standards in the DNA of *Think Math!*" (attached). Having gained proficiency with more than one tool allows students to *choose* realistically among them depending on the situation, rather than defaulting to one method for all situations because that method is the only one they have mastered.

6. **Attend to precision.** *There was moderate evidence found to support development of this practice in the Grades 3 and 4 samples, but little to no evidence was cited in the Grade 5 materials. Evidence of multiple opportunities for communication and examples that model precision were found in the Grades 3 and 4 materials, but in Grade 5, the evidence is lacking.*

Author/Publisher Response:

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently.... In the elementary grades, students give carefully formulated explanations to each other.... —CCSS

Children's use of language varies with development, but typically does not adhere to "clear definition" as much as to holistic images. That is one reason why children who can *state* that a triangle is a closed figure made up of three straight sides may still choose  as a better example of a triangle than  because it conforms more closely to their mental *image* of triangles, despite its failure to meet the definition they gave.

Think Math! is meticulous in its use of mathematical vocabulary and symbols. For example, as soon as students see the = sign, it is used in contexts like 9 + ___ = 8 + 2, focusing on the equality of expressions, not the heralding of an answer. Teacher Guide information about vocabulary is clear and correct, and also helps teachers understand the *role* of vocabulary in clear communication: sometimes fancy words distinguish meanings that common vocabulary does not, and in those cases, they aid precision; but there are also times when fancy words obfuscate, and the goal is always precision and clarity of communication. *Think Math!* does not ask children to "*state* the meaning of the symbols" in any formal way—that is a level of abstraction inappropriate at this age—but does require them to *demonstrate* their understanding of the meaning through unambiguous and clear use.

Think Math! develops vocabulary as it is needed, and where the vocabulary has been listed in standards as a goal, *Think Math!* sets up situations in which an activity of mathematical and intellectual merit on its own *requires* that vocabulary for clear communication. In other words, students acquire the vocabulary not just to show that they have it, but because they genuinely *need* it in order to communicate, in context, about something in which they are engaged. The three dimensional geometry chapters in grades 3, 4, and 5 are particularly good examples of this strategy, though the strategy is used throughout. Where standards, prior to the CCSS, required the terms *vertex*, *edge*, and *face*—and where standards, including the CCSS, require nomenclature for the various quadrilaterals—there was typically little in curricula that one could *do* with these terms except show that one knew what they mean by giving examples or identifying the features. *Think Math!* crafted a diverse set of nets—a different net for each child in class—so that the solids that students create are too different to distinguish by name alone, and must be *described* carefully. Children's natural desire to be able to describe "their own" creation motivates their use of specialized vocabulary such as "my object has two square faces, not the same size, and four faces that are trapezoids." Carefully formulated descriptions are also motivated by puzzles that identify objects only by clues about their features—such as "I have six faces, all rectangular; two of my faces are squares" (which helps children internalize the classification of squares as special rectangles) or "I have exactly as many vertices as faces" (which identifies all the pyramids).

7. **Look for and make use of structure**. *There was minimal evidence of this practice found in the Grades 4 and 5 samples. The practice is more developed in Grade 3, with evidence of prior learning applied to new learning and some examples of making generalizations from observed patterns.*

Author/Publisher Response:

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property.... They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.... —CCSS

Think Math! focuses on *structure* in arithmetic throughout. The Cross Number Puzzle, described in "Common Core State Standards in the DNA of *Think Math!*" and more fully in http://thinkmath.edc.org/index.php/Cross_number_puzzles exemplifies a structure—equals added to equals results in equals—that underlies elementary calculations through the solution of algebraic equations. Students in Grades 4 and 5 (see, e.g., Grade 4, Chapter 14, Lesson 5, and Grade 5, Chapter 2, Lessons 8 through 11) investigate a surprising pattern in multiplication that can enhance their mastery of calculation (as it is treated in Grade 4) and that leads to an interesting generalization (Grade 5—for more, see http://thinkmath.edc.org/index.php/Difference_of_squares).

Students, throughout, are encouraged to reason about calculations before applying brute force to the calculations. Younger children should see 23 + 41 + 27, recognize the 3 + 7, and think 80 + 10 + 1 = 91. Students in Grades 4 and 5 learn to compare the magnitude of fractions by thinking about "distance" from 0, ½, and 1. For example, they reason that 5/7 > 3/5 because the latter is 2/5 less than 1, and the former is only 2/7 less than 1 and thus closer to 1. Similarly, students in *Think Math!* classes are quite often heard explaining, articulately (as called for in Mathematical Practice 6), things like "4/7 and 3/5 are both greater than half, and 3/5 is bigger because it's half of a fifth more [that is, it is *three* fifths rather than *two and a half* fifths, which is exactly ½] and 4/7 is only half of a seventh more." They learn to calculate common denominators, of course, but also learn to reason about the structure. Similarly, in Grade 5, Chapter 4, students investigate fractions in a way that treats the numerator as multiplying and denominator as dividing and shows that these operations can be performed in either order: instead of treating $7 \times {}^{5}/_{42}$ as ${}^{35}/_{42}$, then to be reduced, students learn to look for opportunities to find common factors first.

All of the algebraic thinking (Grade 4, Chapter 14 and Grade 5, Chapters 1 and 13) is approached as structure, not rules. And *Think Math!*'s approach to decimals explicitly uses the image of zooming in on the number line (Grade 4, Chapter 8; Grade 5, Chapter 7) to show the parallel structure of the arithmetic of whole numbers and decimals. Students derive structure from early mental mathematics (Skill Practice and Review) with making pairs that make 10 (e.g., 7,3; 3,7; 8,2; etc.) to complementary pairs to 100 (e.g., 70,30; etc.) to complementary pairs to 1 (e.g., 0.7,0.3). They extend these so that they can use them to recognize that just as 76 is between 70 and 80, with distances of 6 and 4 from those two, respectively, 7.6 is between 7 and 8, with distances of .6 and .4 from those two nearest whole number neighbors. They learn to use this reasoning to "see," mentally, a number line image that lets them compute, e.g., 10.1 – 7.6 in their heads by picturing the distance between these numbers as .4 + 2 + .1, or 2.5.

8. **Look for and express regularity in repeated reasoning.** *There is minimal evidence of this practice in the sampled sections of this series. The teacher notes encourage instructors to ask questions (e.g., "What do you notice? What do you see? Can you describe the pattern?), but generally this practice is underdeveloped throughout this series.*

Author/Publisher Response:

> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.... As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. —CCSS

Mathematics is open to experiment, and general results, or at least conjectures, often spring from trying examples, looking for regularity, and seeking what seem to be general trends. This approach to performing experiments and looking for the regularity pervades *Think Math!* The experiment described above (Gr 4, Ch 14, Lesson 5, and Gr 5, Ch 2, Lessons 8–11, elaborated in http://thinkmath.edc.org/index.php/Difference_of_squares) is just one example. Encoding specific experiments with general language (throughout the program but see, especially, e.g., Grade 14, Chapter 14, all; and Grade 5, Chapter 1, Lessons 3, 5, 6) is yet another example.

The metacognitive skill of "[maintaining] oversight of the process" is systematically and deliberately developed in *Think Math!* The brief description of this approach in http://thinkmath.edc.org/index.php/Developing_attention%2C_focus%2C_and_working_memory gives examples only from the early grades, but students who are learning (e.g., in the context of developing a division algorithm in Grade 5, Chapter 8) to multiply any number by 5 by multiplying by 10 and taking half the result (or, as with any fraction operation, reversing the order and taking half first and then multiplying by 10), must keep clear mental track of where they are. For people who are fluent with the computations, this feels like two steps—not much to keep track of—but for children, multiplying 48×5 mentally involves keeping many numbers in mind. Just to take half of 48 requires holding 48, 40, 8, half of 40, and half of 8 in mind, and selecting the 20 and 4 to make 24. Students must also keep in mind the goal, and the remaining step in order to get 240.

The CCSS Mathematical Practices in *Think Math!*

Mathematical Practices in *Think Math!* pervade the entire program in age-appropriate ways; they are not handled as a separate thread or set of special lessons or supplement. The mathematical habits of mind that we articulated in 1996 became a driving principle in *all* of the EDC curriculum development work, and are central to the way we designed *Think Math!* to help students develop precisely the kind of mathematical practices described in the CCSS. This preliminary document illustrates just a few examples of the ways each of the eight mathematical practices are exemplified in grades 3 through 5. EDC is preparing a more comprehensive document that shows the centrality and depth of Mathematical Practices at *all* the grades.

1. **Make sense of problems and persevere in solving them.**

   Think Math! aims to help students make sense of *all* of the mathematics they learn. One design principle for achieving that is to put experience before formality throughout, letting students explore problems and deriving methods from the exploration. Students in upper grades, for example, learn the logic of multiplication and division—the distributive property that makes possible the algorithms we use—before the algorithms. (Algorithms are taught, but are capstones, not foundations.) The "Explore" features throughout *Think Math!* (see hardbound Student Handbooks in Grades 3-5) exemplify that design principle, but are not the only place it is used. *Think Math!* builds stamina in problem solving both through plentiful experience with problems, but also through a unique, careful, deliberate, and pervasive development of the mathematical "infrastructure" of focus, attention, and working memory. (See http://thinkmath.edc.org/index.php/Developing_attention%2C_focus%2C_and_working_memory.)

   > Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution ... rather than simply jumping into a solution attempt. —CCSS

   To focus squarely on meaning and on analyzing givens and constraints, and to assure that students aren't "simply jumping into a solution attempt," *Think Math!* builds in, *every day of every year*, a kind of problem that asks *only* for the analysis and not for an "answer." This feature, "Headline Stories" (open-ended problem solving) uses several strategies. Sometimes it omits the question, requiring students to analyze the situation and decide what questions would make sense. Sometimes it omits crucial data, requiring students to consider what new information they would need. (See, e.g., Grade 4, TG p. 1061.) Of course, *Think Math!* also provides plentiful non-standard problems of the familiar variety. (See, e.g., Grade 4, TG p. 1068.)

   > Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, ... Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. —CCSS

   Mathematically powerful representations including the number line, area model, and tabular/spreadsheet forms are taught, as a kind of common language, to students. They use these appropriately and *show* their understanding of the correspondence between symbolic forms and other representations. *Think Math!* also supports the development of appropriate academic and mathematical language—information for the teacher is supplied in the TG for each lesson—and encourages question and discussion in class.

2. **Reason abstractly and quantitatively.**

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. —CCSS

In Grade 4, chapter 3, students are exploring regrouping in a new way in preparation for formalizing their understanding of place value for computation. Children work with a context, an eraser factory that groups 7 erasers to a package, 7 packages to a box. They *decontextualize* as they study their shipments to assure the no-more-than-6 rule is observed; and they *contextualize* to make sense of the trades and to work out how many erasers in a particular size container. They must constantly be attending to the units involved and the meaning of quantities. The choice of 7 practices their 7-facts, but the students' focus is on regrouping.

3. **Construct viable arguments and critique the reasoning of others.**

> ... Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. ... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. —CCSS

Think Math! students are involved, in all grades, in age-appropriate proof and justification—as means for explaining what they have done, seen, or figured out. Grade 5, Chapter 2, lessons 8-11 are a particularly good illustration. Students perform a multiplication experiment from which they derive an unexpected pattern—the square of a number (e.g., 8×8) will always be one greater than the product of that number's neighbors (e.g., 7×9)—they are challenged to describe the pattern, extend the pattern, and even investigate a proof (using the area model) for the pattern. Students develop posters that demonstrate their proof, and explain their understanding to each other.

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4. **Model with mathematics.**

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life.... In early grades, this might be as simple as writing an addition equation to describe a situation. .... Mathematically proficient students who can apply what they know are

> comfortable making assumptions and approximations to simplify a complicated situation.... They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs.... They...reflect on whether the results make sense.... —CCSS

The problem "Sam Houston Elementary School has nearly 1,000 children from kindergarten through 5th grade, with about the same number of students in each grade. Most classes have close to 25 students. No class has more than 25 students. What can you figure out from this information?" appears in grade 5, TG pg 647. It asks students to work with approximations, identify the important quantities and figure out their relationships, make sense (standard 1) and check whether their results make sense. Problems of this kind, age-appropriate complexity and mathematical demand, are quite like the problems of real life, with some loose ends, and not necessarily a clear starting place. They, along with more familiar problem types, appear throughout *Think Math!*

Most of the work of Headline Stories is aimed at having students find and express the mathematics in a situation where information is given, but the question is not. E.g., Grade 4 Chapter 3 Lesson 3 (TG161) "Jin has balancing scales and weights of 1, 2, 4, and 8 ounces." If a targeted question were provided—like "What weights would she use to balance 11 ounces of butter?"—students would get arithmetic practice, but nothing more: as soon as the solution is found, it's all over. A looser question "What quantities of butter can she weigh exactly using one or more of these weights?" gives more practice, and more opportunities to think and puzzle. Over time, students get used to the generic questions: What (mathematical statements) can you say *for sure* about this situation? What mathematical observations can you make? What mathematical questions does this raise? What can you figure out?

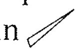
5. **Use appropriate tools strategically.**

> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet.... Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. —CCSS

The variety of tools that *Think Math!* students acquire and become proficient with—including visual models such as number line or area models, mental work, or paper and pencil to support calculations—is described in the paper "Common Core State Standards in the DNA of *Think Math!*" (attached). Having gained proficiency with more than one tool allows students to *choose* realistically among them depending on the situation, rather than defaulting to one method for all situations because that method is the only one they have mastered.

6. **Attend to precision.**

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently.... In the elementary grades, students give carefully formulated explanations to each other.... —CCSS

Children's use of language varies with development, but typically does not adhere to "clear definition" as much as to holistic images. That is one reason why children who can *state* that a triangle is a closed figure made up of three straight sides may still choose  as a better example of a triangle than  because it conforms more closely to their mental *image* of triangles, despite its failure to meet the definition they gave.

Think Math! is meticulous in its use of mathematical vocabulary and symbols. For example, as soon as students see the = sign, it is used in contexts like 9 + ____ = 8 + 2, focusing on the equality of expressions, not the heralding of an answer. Teacher Guide information about vocabulary is clear and correct, and also helps teachers understand the *role* of vocabulary in clear communication: sometimes fancy words distinguish meanings that common vocabulary does not, and in those cases, they aid precision; but there are also times when fancy words obfuscate, and the goal is always precision and clarity of communication. *Think Math!* does not ask children to "*state* the meaning of the symbols" in any formal way—that is a level of abstraction inappropriate at this age—but does require them to *demonstrate* their understanding of the meaning through unambiguous and clear use.

Think Math! develops vocabulary as it is needed, and where the vocabulary has been listed in standards as a goal, *Think Math!* sets up situations in which an activity of mathematical and intellectual merit on its own *requires* that vocabulary for clear communication. In other words, students acquire the vocabulary not just to show that they have it, but because they genuinely *need* it in order to communicate, in context, about something in which they are engaged. The three dimensional geometry chapters in grades 3, 4, and 5 are particularly good examples of this strategy, though the strategy is used throughout. Where standards, prior to the CCSS, required the terms *vertex*, *edge*, and *face*—and where standards, including the CCSS, require nomenclature for the various quadrilaterals—there was typically little in curricula that one could *do* with these terms except show that one knew what they mean by giving examples or identifying the features. *Think Math!* crafted a diverse set of nets—a different net for each child in class—so that the solids that students create are too different to distinguish by name alone, and must be *described* carefully. Children's natural desire to be able to describe "their own" creation motivates their use of specialized vocabulary such as "my object has two square faces, not the same size, and four faces that are trapezoids." Carefully formulated descriptions are also motivated by puzzles that identify objects only by clues about their features—such as "I have six faces, all rectangular; two of my faces are squares" (which helps children internalize the classification of squares as special rectangles) or "I have exactly as many vertices as faces" (which identifies all the pyramids).

7. Look for and make use of structure.

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property.... They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.... —CCSS

Think Math! focuses on *structure* in arithmetic throughout. The Cross Number Puzzle, described in "Common Core State Standards in the DNA of *Think Math!*" and more fully in http://thinkmath.edc.org/index.php/Cross_number_puzzles exemplifies a structure—equals added to equals results in equals—that underlies elementary calculations through the solution of algebraic equations. Students in Grades 4 and 5 (see, e.g., Grade 4, Chapter 14, Lesson 5, and Grade 5, Chapter 2, Lessons 8 through 11) investigate a surprising pattern in multiplication that can enhance their mastery of calculation (as it is treated in Grade 4) and that leads to an interesting generalization (Grade 5—for more, see http://thinkmath.edc.org/index.php/Difference_of_squares).

Students, throughout, are encouraged to reason about calculations before applying brute force to the calculations. Younger children should see 23 + 41 + 27, recognize the 3 + 7, and think 80 + 10 + 1 = 91. Students in Grades 4 and 5 learn to compare the magnitude of fractions by thinking about "distance" from 0, ½, and 1. For example, they reason that 5/7 > 3/5 because the latter is 2/5 less than 1, and the former is only 2/7 less than 1 and thus closer to 1. Similarly, students in *Think Math!* classes are quite often heard explaining, articulately (as called for in Mathematical Practice 6), things like "4/7 and 3/5 are both greater than half, and 3/5 is bigger because it's half of a fifth more [that is, it is *three* fifths rather than *two and a half* fifths, which is exactly ½] and 4/7 is only half of a seventh more." They learn to calculate common denominators, of course, but also learn to reason about the structure. Similarly, in Grade 5, Chapter 4, students investigate fractions in a way that treats the numerator as multiplying and denominator as dividing and shows that these operations can be performed in either order: instead of treating $7 \times {}^{5}/_{42}$ as ${}^{35}/_{42}$, then to be reduced, students learn to look for opportunities to find common factors first.

All of the algebraic thinking (Grade 4, Chapter 14 and Grade 5, Chapters 1 and 13) is approached as structure, not rules. And *Think Math!*'s approach to decimals explicitly uses the image of zooming in on the number line (Grade 4, Chapter 8; Grade 5, Chapter 7) to show the parallel structure of the arithmetic of whole numbers and decimals. Students derive structure from early mental mathematics (Skill Practice and Review) with making pairs that make 10 (e.g., 7,3; 3,7; 8,2; etc.) to complementary pairs to 100 (e.g., 70,30; etc.) to complementary pairs to 1 (e.g., 0.7,0.3). They extend these so that they can use them to recognize that just as 76 is between 70 and 80, with distances of 6 and 4 from those two, respectively, 7.6 is between 7 and 8, with distances of .6 and .4 from those two nearest whole number neighbors. They learn to use this reasoning to "see," mentally, a number line image that lets them compute, e.g., 10.1 – 7.6 in their heads by picturing the distance between these numbers as .4 + 2 + .1, or 2.5.

8. Look for and express regularity in repeated reasoning.

> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.... As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. —CCSS

Mathematics is open to experiment, and general results, or at least conjectures, often spring from trying examples, looking for regularity, and seeking what seem to be general trends. This approach to performing experiments and looking for the regularity pervades *Think Math!* The experiment described above (Gr 4, Ch 14, Lesson 5, and Gr 5, Ch 2, Lessons 8–11, elaborated in http://thinkmath.edc.org/index.php/Difference_of_squares) is just one example. Encoding specific experiments with general language (throughout the program but see, especially, e.g., Grade 14, Chapter 14, all; and Grade 5, Chapter 1, Lessons 3, 5, 6) is yet another example.

The metacognitive skill of "[maintaining] oversight of the process" is systematically and deliberately developed in *Think Math!* The brief description of this approach in http://thinkmath.edc.org/index.php/Developing_attention%2C_focus%2C_and_working_memory gives examples only from the early grades, but students who are learning (e.g., in the context of developing a division algorithm in Grade 5, Chapter 8) to multiply any number by 5 by multiplying by 10 and taking half the result (or, as with any fraction operation, reversing the order and taking half first and then multiplying by 10), must keep clear mental track of where they are. For people who are fluent with the computations, this feels like two steps—not much to keep track of—but for children, multiplying 48 × 5 mentally involves keeping many numbers in mind. Just to take half of 48 requires holding 48, 40, 8, half of 40, and half of 8 in mind, and selecting the 20 and 4 to make 24. Students must also keep in mind the goal, and the remaining step in order to get 240.

Common Core State Standards in the DNA of Think Math!

The first edition of *Think Math!* was published four years before the final draft of the Common Core State Standards for mathematics was first released in June 2010. You might think we'd now have to play catch-up. In fact, that is not the case.

While some *content* details of the CCSS could not possibly have been known before these standards were released, two principles central to the design of the Common Core State Standards in Mathematics were already widely accepted. The first was that mathematical proficiency requires two things: fluent knowledge—the facts, vocabulary, procedures, formulas, and theorems that we associate with math courses—and the habits of mind that allow mathematicians (and others) to generate that knowledge. The second principle was that to build mathematical proficiency, courses needed greater focus. The many disconnected bits of the "mile-wide, inch-deep" curricula that had evolved over the years gave students neither the time to develop comfort and skill with key ideas and practices nor the coherence to make sense of those ideas and assemble them into a mental discipline.

These two concerns were part of the very DNA of *Think Math!*, driving principles behind its design from the time we first proposed it to the National Science Foundation in 1999. In fact, our concern about the mathematical thinking that the CCSS now mandates in its Mathematical Practice standards had been central to our work at EDC even earlier. We published "Habits of mind, an organizing principle for mathematics curriculum" in 1996 after roughly four years developing and applying those ideas. The ideas we first articulated in 1996 and extended in many subsequent articles have found a home in the CCSS Mathematical Practices.

Our concern for finding ways to bring focus and depth to curricula—despite the pre-CCSS plethora of topics required by disparate state frameworks—was a consequence of our determination to build both the *skills* and the *thinking* of mathematics. *Think Math!* was designed around a highly focused set of curricular goals, but achieved those goals through a mathematically rich and diverse set of examples. Not only *could* mathematical contexts vary richly while emphasizing a tightly focused set of goals, they *had* to vary in order to develop depth of understanding and in order to make the necessary practice intellectually interesting.

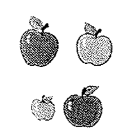
These two concerns also drove the thinking behind the CCSS. To help students develop the mathematical proficiency they need not only for science, technology, engineering, and math but also for auto diagnosis and repair, medical diagnosis and treatment, social science research and business, architecture, construction, and the management of one's own money in an age of financial complexity—virtually all aspects of modern American life—curricula and instructional plans had to teach students to think mathematically, and also had to have sufficient depth and focus to help students gain skill and proficiency. Accordingly, the CCSS presents two kinds of standards: Standards for Mathematical Content and Standards for Mathematical Practice. The content standards are listed in a way that is, in its *form*, not unlike lists we've seen in state frameworks for years, but they are more lean and focused, and represent "a balanced combination of procedure and understanding" with a mandate to connect the practices to the content. Despite the long history of thought behind them, the practice standards are the "new" element, less familiar to most people. They outline critical aspects of mathematical thinking: "...ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise..." (CCSS, 8).

Mathematical Practices

The eight Mathematical Practice standards complement the content standards by aiming curricula, teaching, and tests at helping students to see their world through a mathematical lens, reason mathematically, and *use* mathematics thoughtfully and effectively to solve problems. All eight are addressed throughout *Think Math!*, some on a nearly daily basis. I will illustrate that with two of them.

1. Make sense of problems and persevere in solving them. The CCSS describes this standard by saying that "mathematically proficient students ... [look] for entry points... analyze givens, constraints, relationships, and goals. ... They monitor and evaluate their progress and change course if necessary." A major part of problem-solving is figuring out how to begin. Real-life problems don't ask what chapter you're in; they just appear. And you must figure out what information you have, what you need, where to start, and what to do. Whether you are an engineer, a teacher, a doctor, an auto mechanic, or a mathematician, you always face new situations for which you must puzzle out where to begin and what action to take, and then watch to see how it changed the situation, and what might be sensible to do next.

In *Think Math!*, this kind of sense-making and puzzling through is part of students' everyday experience. Most of the Headline Stories ask students to analyze the givens and make sense of them, deriving new mathematically relevant statements or posing new questions about the situation. Kindergarteners discussing this picture will describe it as four apples, or three big apples and one little one, or two red and two green, beginning the process of learning that more than one mathematically relevant statement can often be made about a single situation. (More about Headline Stories at http://thinkmath.edc.org/index.php/Headline_stories.)

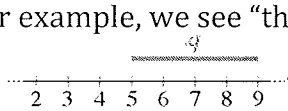


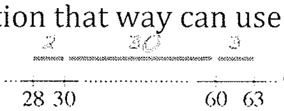
The Cross Number Puzzle is another way in which *Think Math!* integrates content, mathematical representation, and reasoning. In this puzzle, the sum of the numbers on one side of the heavy line in any row or column must equal the sum of the numbers on the other side of that heavy line in the same row and column. The *content* is the logic of addition and subtraction and the role of place value in calculating using the standard algorithms. The *representation* is a table, a mini-spreadsheet. This table organizes kindergarteners' sorting, second graders' adding and subtracting, and fifth graders' division, and its logic prepares students for reasoning about systems of equations when they reach algebra. But the reason we present this tabular form *as a puzzle*, aside from pure appeal, is that the nature of any good puzzle is about "looking for entry points," figuring out how to start and what to do next. Thus, grade-appropriate elementary school content is presented in a way that reveals an underlying mathematical structure that prepares students for algebra, and in a form

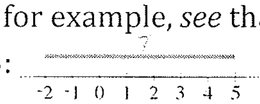
	4	
10		16
20		

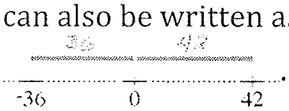
that evokes and teaches an essential problem-solving process. (For more about Cross Number Puzzles, see http://thinkmath.edc.org/index.php/Cross_number_puzzles.)

5. Use appropriate tools strategically. The CCSS makes clear that these tools are not just physical tools such as "pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet..." but also mental tools such as "estimation and other mathematical knowledge." Along with one of the most powerful mental tools of all—the puzzler's disposition of first assessing what one knows and thinking strategically about where to begin—*Think Math!* provides three particularly powerful mathematical tools that are appropriate in the earliest grades and remain valuable throughout high school mathematics and beyond. One, the early and continued emphasis on tables and, in particular, the spreadsheet-like table we call the Cross Number Puzzle, has already been mentioned.

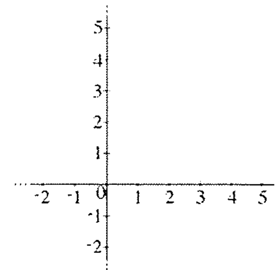
The second important tool, the number line, is sometimes regarded just as a visual aid for children; it is, in fact, a sophisticated image used even by mathematicians. For young children, it helps make early mental images of addition and subtraction that connect arithmetic with measurement—rulers, which students will also get plenty of chance to use, are just number lines built to spec! Here, for example, we see "the distance from 5 to 9" or "how much greater 9 is than 5" as 

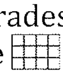
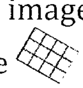
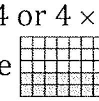
. Children who see subtraction that way can use this model to begin to see "the distance between 28 and 63" as 35 

. Many learn to see that in their heads, too, and do this subtraction mentally. This is essentially how clerks used to "count up" to make change. Because the number line model extends so naturally to decimals and fractions—just by "zooming in" to get a more detailed view of that line between the whole numbers—and extends equally naturally to negative numbers, it *unifies* arithmetic, making sense of what is otherwise often seen as a collection of independent and hard-to-remember rules. We can, for example, *see* that the distance from ⁻2 to 5 is the number we must add to ⁻2 to get 5: 

. And we can *see* why 42 – ⁻36 can also be written as 42 + 36: the "distance from ⁻36 to 42 is 

. The number line remains useful as students study data, graphing, and algebra: two number lines, at right angles to each other, label the addresses of points on the coordinate plane.



The area model of multiplication is another powerful tool that lasts from early grades through college mathematics. For second graders in *Think Math!*, images like  along with questions like "how many columns, how many rows, how many little squares" help establish the "small" multiplication facts. So might pure drill, but this image goes much farther. Seeing the same array held in different positions like  and  makes clear that we can label any of these 3 × 4 or 4 × 3 and the number of little squares is always 12. In grades 3-5, array pictures like  help clarify the distributive property of multiplication, the property that makes multi-digit multiplication possible, makes sense of the standard multiplication and division algorithms, and underlies the multiplication that students will encounter in algebra. In this picture, we see that "two 7s plus three 7s is five 7s" or $2\times7 + 3\times7 = (2 + 3) \times 7$. A schematic version of this image organizes students' thinking as they learn multi-digit multiplication; this image, combined with the Cross Number Puzzle, models the conventional algorithm exactly, making total sense of what can otherwise feel like an arbitrary set of steps. The same image also allows students to acquire and understand the algorithm for division as a process of "undoing" multiplication, greatly simplifying the learning of a part of arithmetic that has a long history of being difficult. What makes this a *powerful* tool is that it serves the immediate goals of elementary school arithmetic in a way that prepares students for algebra. Algebraic multiplication, which has no "carry" step, is modeled perfectly by exactly the same tool. (To learn more about how *Think Math!* develops the use of the area model tool, see http://thinkmath.edc.org/index.php/Multiplication and http://thinkmath.edc.org/index.php/Multiplication_and_division.)

This versatile suite of tools builds *mental* models that last. What makes a tool like the number line, area model, and table/spreadsheet truly powerful is that it is not just a special-purpose trick or temporary crutch, but is faithful to the mathematics and is extensible and applicable to many domains. These tools help students make sense of the mathematics; that's *why* they last. And that is also why the CCSS mandates all three of them.

Of course, *Think Math!* also helps students make strategic use of *physical* tools. When students first study decimals, they do it with a "native speaker of decimal," the calculator. The calculator does not *replace* the students' thinking; as a native speaker, it helps them learn the new language, the decimal notation.

And students learn paper and pencil methods, as well. Because the standard algorithms are efficiently honed, compact notations, they hide the logic students need to learn for algebra. While that makes them poor ways of *acquiring* the logic of computation, that compactness is also their great virtue *after* one understands the logic. By reducing the "cognitive load"—the amount of thinking we need to do in order to perform a calculation—they raise speed and accuracy for calculations that we cannot quite keep entirely in our heads and that we don't do with a calculator. So these are taught, but not as "how to calculate" but as a *summary*, a compact notation, for the processes students have already acquired logically. That way, students may choose between mental methods, paper and pencil, and calculator, rather than defaulting to the paper and pencil method as the only "legitimate" way, or the calculator as the only easy way.

Content Standards: Skill and Understanding
(teaching focused content through mathematically rich and diverse contexts)

> The Standards for Mathematical Content are a balanced combination of procedure and understanding. (CCSS, 8)

Popular debate has often pitted skill and understanding against one another, as if time spent on one steals time from the other. But this is not the case. No tradeoff is required and, in fact, it is not generally *possible* to have one without the other.

There are, of course, facts and skills that you acquire without "understanding." Your friends' names are such facts, and walking is such a skill; both get constant enough use that nothing else in needed to sustain them. Salience—from constant use, from a sense that it is important, or just because it interests you—is what makes a fact stick. For fluid skill—in mathematics, violin, or skating—practice is also essential. Some students find the details of, say, long division sufficiently salient that they can acquire and retain that skill without knowing why it works. But many students cannot and, even for those who can, the skills are difficult to maintain without conceptual scaffolding to prop them up. Rules that seem arbitrary are very hard to remember correctly.

It is equally possible to develop certain kinds of understanding without skill—for example, you can understand roughly the way a car works without knowing any of the fine details or having any skill at fixing it—but mathematical understanding is hard to develop, quick to fade, and nearly impossible to apply without skill. You can't recognize a numerical pattern if you don't already have in your head the fluent knowledge that allows you to see how the numbers are related; you can't see the underlying sense of a mathematical procedure if all of your attention is taken up just with managing the details. And, besides, the very purpose of "understanding mathematics" is to help you *do* things with it.

How do you build these two at the same time? Both *Think Math!* and the CCSS, at different times, did it the same way. The CCSS chose content strategically and calls for connecting that content with mathematical practices. The *Think Math!* designers aimed for almost exactly the same content—with very few exceptions—and succeeded in focusing on those topics *despite* having to meet state standards that mandated a more diverse set of topics by delivering *some* of the topics we considered less central *through* practice or application of the central ones. And, by *organizing* the curriculum around big mathematical ideas—not isolated topics, but pervasive themes, central properties, mathematical habits of mind, and what the CCSS calls Mathematical Practices—"distinct" topics would support each other rather than compete with each other for time. For example, area is listed in the CCSS under "Measurement," but the CCSS explicitly calls for it to be tied to ideas about multiplication and the distributive property. That was exactly the way *Think Math!* designed both topics to connect.

Handling one topic in a way that serves another is also how *Think Math!* can stay completely focused on elementary school mathematics—focused and *deep*—and yet do it in a way that prepares students well for the algebra that they will learn in later grades. Even the Mental Math (Skill Practice and Review) exercises that we provide—essentially drills—are strategically designed for that purpose. Random drill focuses only on rote memory; *Think Math!*'s mental math exercises build a kind of intuitive sense of all of the central properties of arithmetic. Doubling and halving everything practices mental use of the distributive property; multiplying by 5 by multiplying by 10 and then halving practices the associative property of multiplication; pairs to 10 and to 100 and (later, with decimals and fractions) to 1 practice complements and the ability to maintain a mental number line. Students who get this kind of practice develop great facility with mental computation, but they get way more than that: they build "gut" familiarity with all of the properties of arithmetic that they will need for algebra.

Handling content in ways that focused on the mathematical ideas and practices was a particular challenge as we were designing way to meet the prior frameworks' calls for a vast set of seemingly arbitrary geometric vocabulary. We could just teach it, of course—children can memorize those words as easily as they can memorize their classmates' names—but teaching disconnected terms felt anti-mathematical and, besides, took time away from core content we (and later CCSS) considered core. So we took on the fascinating challenge of inventing a mathematically significant activity—something that served more central goals of *Think Math!* and was interesting to children—that would naturally *need* those terms for clear communication. (Using words in communicative context is, anyway, how children acquire vocabulary best and how they've acquired the vast bulk of their vocabulary which, by first grade, is already half what their adult vocabulary is likely to be.) In our 3-D geometry chapters, each student in the class folds a unique net, making a very diverse set of 3-D shapes, as many different ones as there are children in the class. Because the set *is* so diverse, the individuals are not trivial to name; describing your own "creature" requires some careful analysis of its features. Naming the number and shape of the faces practices 2-D shape vocabulary. Making other observations and inventing puzzles about the shapes requires new 3-D vocabulary. Counting vertices, edges, and faces is no longer an arbitrary requirement, but part of a well-motivated exploration of the nature of prisms, pyramids, and polyhedral objects that are neither of those, making some intriguing discoveries, and communicating all of that clearly. The CCSS (Mathematical Practice 6) says "Mathematically proficient students try to communicate precisely to others." Our activity guarantees not only that students *have* usefully precise descriptive ideas and vocabulary, but also have the motivation to be eager to communicate that to others—as the CCSS puts it, to "*try* to communicate precisely." The fact that each shape is unique, and that each child "owns" a shape, puts a great premium on being able to *describe* your own shape with precision. We also try to make clear to the teacher, as well as to the student, what is (and even what isn't) essential about the vocabulary. (To see how *Think Math!* treats some mathematical terms, see http://thinkmath.edc.org/index.php/Face, http://thinkmath.edc.org/index.php/Width, and http://thinkmath.edc.org/index.php/Right_angle.)

These references, in order by date, are a selection from our work on mathematical habits of mind and the design of curriculum:

Cuoco, A., Goldenberg, E. P., & J. Mark. (1996). Habits of mind: an organizing principle for mathematics curriculum. *Journal of Mathematical Behavior*. **15**(4): 375-402.

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Schifter, D., Bastable, V., & S. Russell. (1999). *Developing Mathematical Ideas* (DMI). Parsippany, NJ: Dale Seymour Publications.

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Cuoco, A. & E. P. Goldenberg. (2010). *Mathematical habits of mind in the Common Core State Standards.* Retrieved from http://ttalgebra.edc.org/index.php/Algebraic_habits_of_mind.

Cuoco, A., Goldenberg, E. P., & J. Mark. (2010). Organizing a curriculum around mathematical habits of mind. *Mathematics Teacher* **103**(9): 682-688.

Goldenberg, E. P., Mark, J., & A. Cuoco. (2010). An algebraic-habits-of-mind perspective on elementary school. *Teaching Children Mathematics* **16**(9): 548-556.

Mark, J., Cuoco, A., Goldenberg, E. P., & S. Sword. (2010). Developing mathematical habits of mind. *Mathematics Teaching in the Middle School* **15**(9): 505-509.

School Specialty Math, Think Math!

Comments

Grade 3

School Specialty Math, **Think Math!** added a Common Core Resource Guide as a way to more closely align to the Common Core State Standards. **Think Math!** is a research-based program, funded in part by the National Science Foundation, and great care was taken in adding lessons and activities to the program. In November of 2010, we submitted a draft of the proposed Common Core Resource Guide to the IDOE. This guide included a sample lesson from Grade 2. In January of 2011, as per instructions, we sent the final individual K-5 grade-level Common Core Resource Guides to the IDOE. Unfortunately, these completed guides were not reviewed as part of the **Think Math!** program.

Some of the standards received a 0 rating by the committee. These standards are covered in the Common Core Resource Guide. In Grade 3, you will find the following additions to the program. As you can see by the descriptions below, the Common Core Resource Guide lesson and activity additions adequately cover the Common Core State Standards. We have included these lessons and activities in the binder.

It is also important to add that in grades 3, 4, and 5 the core student materials are the Lesson Activity Book, the hardbound Student Handbook, the Practice Book, and the Common Core Resource Guide. The Practice Book is used based on the teacher's ongoing assessment of the class. It appears that a reviewer, particularly grade 5, referred only to the Student Handbook.

Common Core State Standards 3.NF.2 and 3.NF.3
Chapter 7: An additional lesson, 6-1, *Fractions on a Number Line,* has been added to Chapter 7.
About the Lesson:
In previous lessons, students have been thinking about a fraction as a part of a whole. This lesson presents another way to think about a fraction--as a number on a number line. For example, ½ is a number halfway between 0 and 1. Students will see that, when talking about fractions as numbers, equivalent fractions are at the same location on the number line. The lesson includes Developing Mathematical Language, Headline Story, Skills Practice and Review, and Teach and Practice. Differentiated Instruction is also included in the complete new lesson as it is in all **Think Math!** lessons.

Common Core State Standard 3.MD.3
Chapter 8: The activity *Making Bar Graphs* in Chapter 8, Lesson 3, *Introducing Bar Graphs,* has been extended.
About the Activity:
In this extended activity, students make a bar graph from survey data and use the information from the graph to solve one- and two-step word problems.

Common Core State Standard 3.MD.4
Chapter 10: An additional activity, *Making a Line Plot,* has been added to Chapter 10, Lesson 2, *Measuring Heights.*
About the Activity:
In this added activity, students are introduced to line plots, and use the data collected about their standing and seated heights to draw a line plot. This lays the foundation for students' study of bar graphs in lesson 10.3.

Common Core State Standard 3.MD.2
Chapter 13: An additional lesson, 5-1, *Measuring Mass in Grams and Kilograms*, has been added to Chapter 13.
About the Lesson:
In previous lessons, students have explored weight in ounces, pounds, and tons. This lesson focuses on using the metric units of grams and kilograms to measure mass. Students learn benchmarks for these units and recognize that a kilogram is a lot heavier than a gram. They learn that 1 kilogram is equal to 1,000 grams. The lesson includes Developing Mathematical Language, Headline Story, Skills Practice and Review, and Teach and Practice. Differentiated Instruction is also included in the complete new lesson as it is in all **Think Math!** lessons.

Common Core State Standard 3.MD.2
Chapter 13: The activity *Measuring with Liters* in Chapter 13, Lesson 6, *Measuring Capacity,* has been extended.
About the Activity:
In this extended activity, students are introduced to the liter. Then students estimate and measure liquid volumes in liters.